

NAPIER'S BONES -- A tool to do multiplication problems

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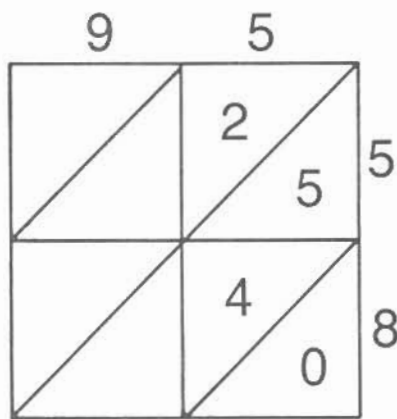
Step 1: Fill in the multiplication facts

Step 2: Cut in strips (the arrows show where to cut)

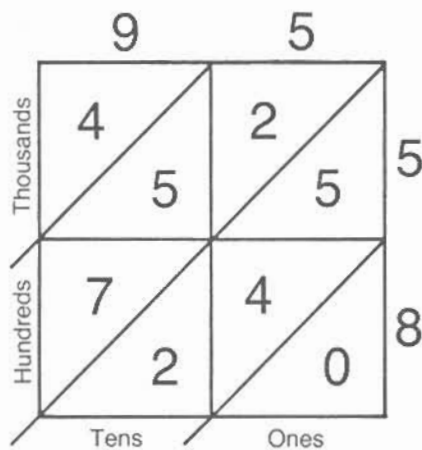
Lattice Multiplication and Division

By Frank W. Broadbent

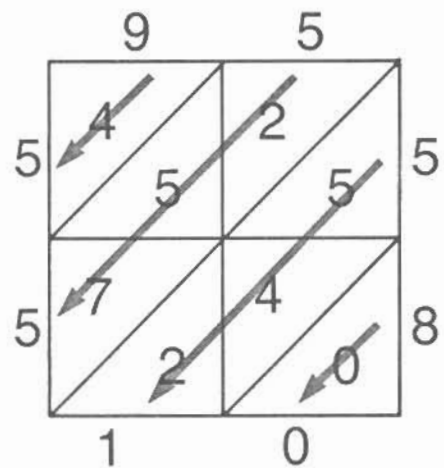
Fig. 1 Multiply 95×58 .



(a) Record the results of 5×58 .



(b) Record the results of 9×58 .



(c) Add diagonally; beginning at lower right, to find the result, 5510.

We often find solutions to new problems in older methods. This pattern may hold for the use of a newly revised algorithm for multiplication and division. The original historical algorithm was the lattice method of multiplication used in Europe as early as the fifteenth century and later made more useful by John Napier and his bones in the sixteenth century. Lattice algorithms are easy to teach, and students are fairly successful with

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them. Unfortunately, they require more time to teach than our standard algorithms. Now that any calculation requiring a significant amount of time is done on calculators or computers, the present standard algorithms, which are highly efficient but difficult to teach and prone to error, need to be reconsidered.

The standard algorithms for multiplication and division do not indicate the inverse relationship of these operations. A modern adaptation of the lattice algorithm, which can be used for both multiplication and division, clearly shows this inverse relationship. The new algorithm incorporates elements of Hutchings's (1976) low-

stress algorithms. The original lattice is placed on the multiplicand across the top; the multiplier is placed down the side of a lattice composed of cells with single diagonals separating the cells into tens and ones. Multiplying 95×58 would be set up like figure 1 with products of digits written in tens and ones in the appropriate cells. The first column is $5 \times 5 = \boxed{25}$ and $5 \times 8 = \boxed{40}$ (fig. 1(a)). The lattice in figure 1(b) has all the products written in and the place values labeled. Adding down each diagonal starting at the lower right corner and carrying to the next diagonal results in the product 5510, as shown in figure 1(c).

To make regrouping easier, we can include narrow columns, or strips, to accept the numbers that are carried. The same problem is solved in figure 2 using the altered algorithm. Notice that *as soon as* a sum exceeds 9, the 1 to be carried is placed in the next higher strip.

The decimal point in a product can be located by placing it in the correct row and column, then in a diagonal, as illustrated in figure 3. No counting is necessary.

This lattice can be modified for division by adding a narrow strip to the left or right to record remainders and by adding another strip across the bottom to assist in regrouping and borrowing. Figure 4(a) illustrates the division of 34 029 by 87. First set up the lattice (fig. 4(a)). Start at the left, dividing 340 by 87. A "rounded up" estimate is used, making the 87 a trial divisor of 90 and the trial quotient a 3. The products of 3 and 87 are then placed in the first column of cells in figure 4(b).

Starting at the bottom left, we see that 1 cannot be subtracted from the 0 in 340. Therefore, one of the four tens

in 340 is exchanged for ten ones and placed in the horizontal strip. Then 1 is subtracted from 10, and the remainder, 9, is placed in the vertical strip at the right. A 3 for three tens is placed in the left vertical strip. The 4 and 2 in the next diagonal are added to make a

subtrahend of 6. Since 6 cannot be subtracted from 3, ten tens are borrowed from the three hundreds in 340 and a 2 for 200 is recorded in the top of the vertical strip on the left. Six (4 + 2) subtracted from 13 leaves 7, which is placed in the vertical strip

Fig. 2 Alter the lattice design to include strips for carrying.

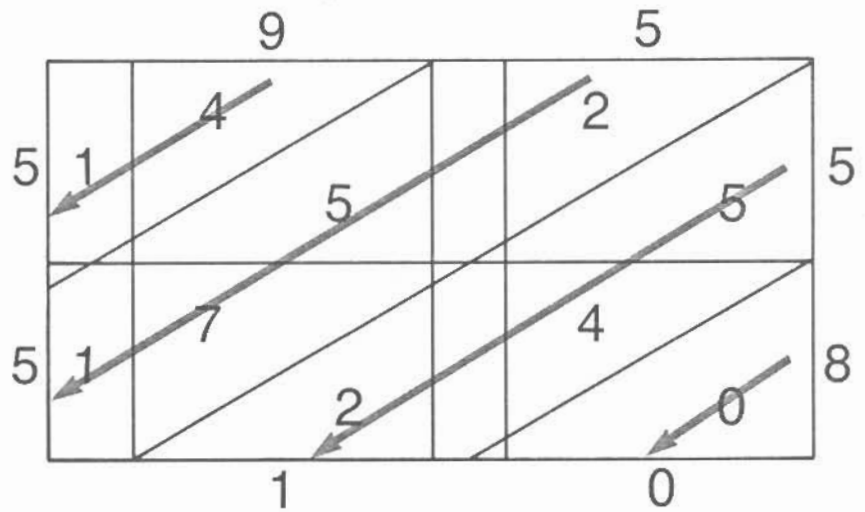


Fig. 3 $1.86 \times 72.6 = 135.036$.

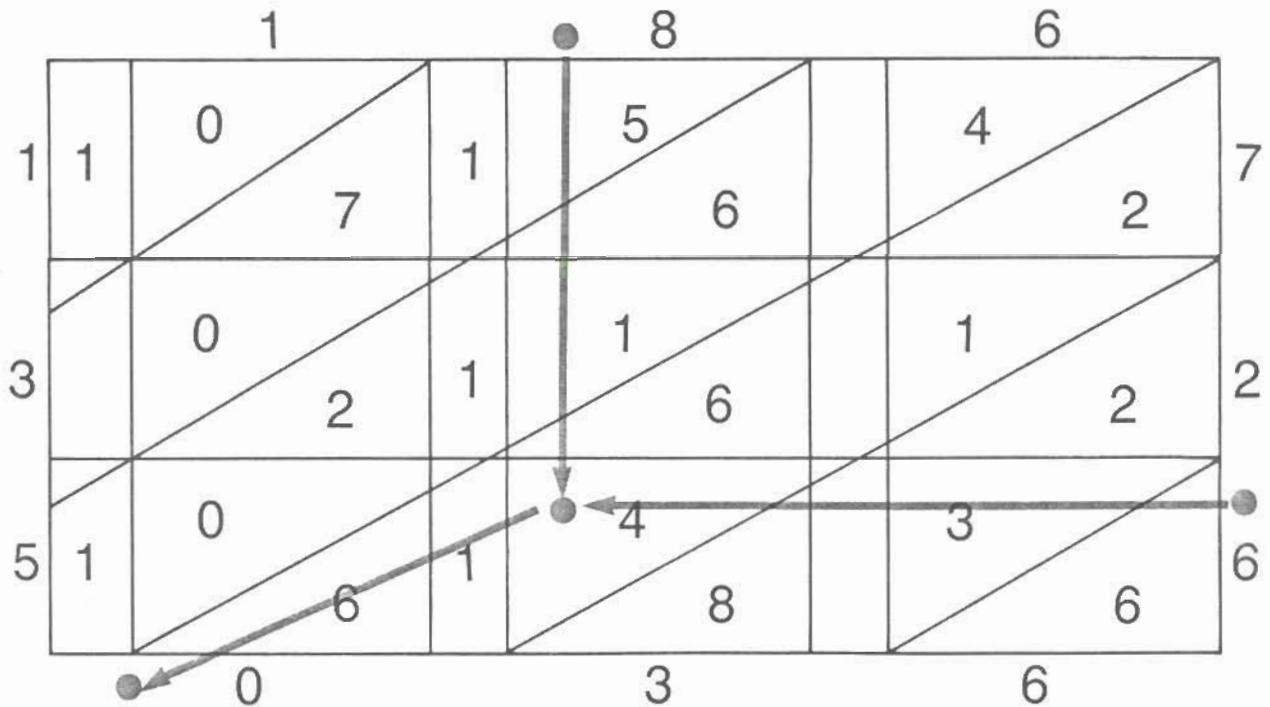
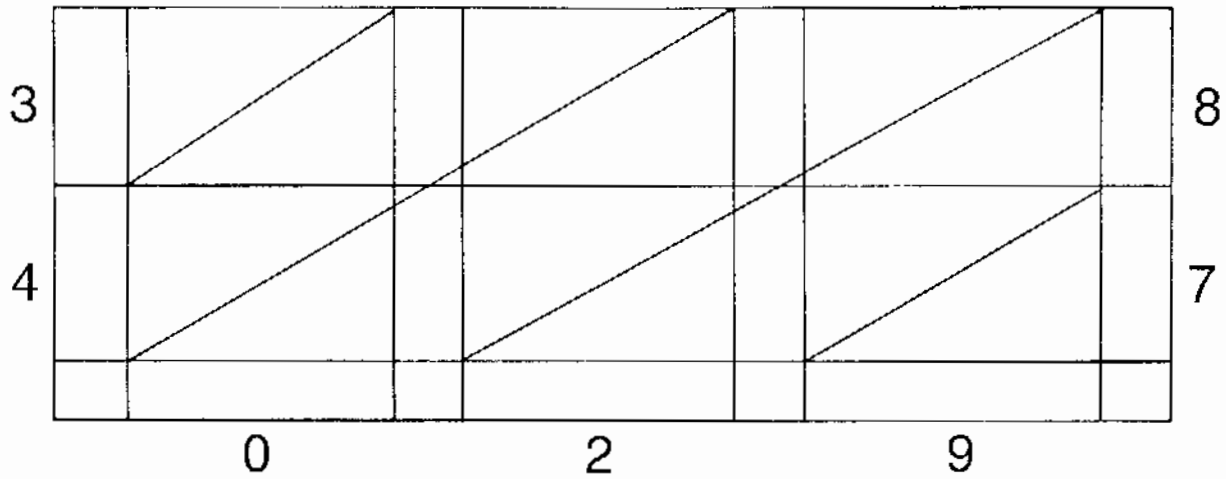
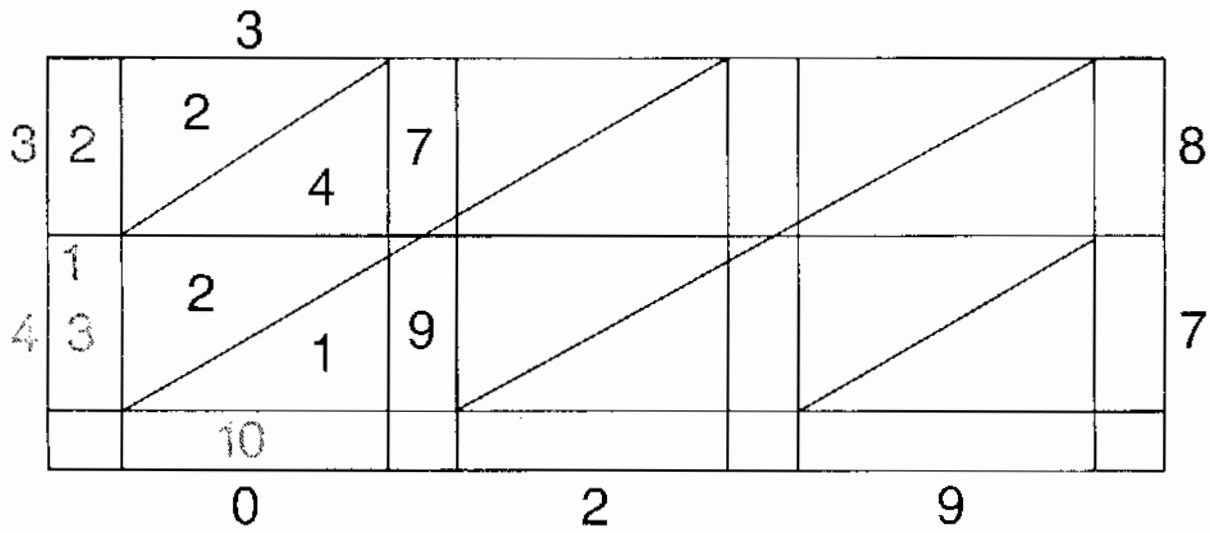


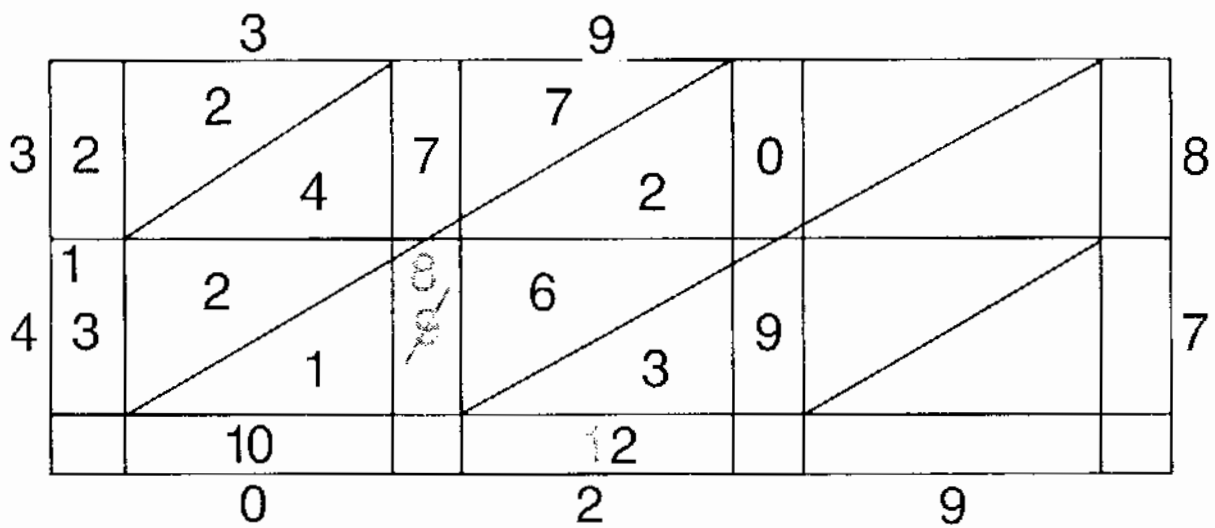
Fig. 4 Divide 34 029 by 87.



(a) Set up a division lattice.

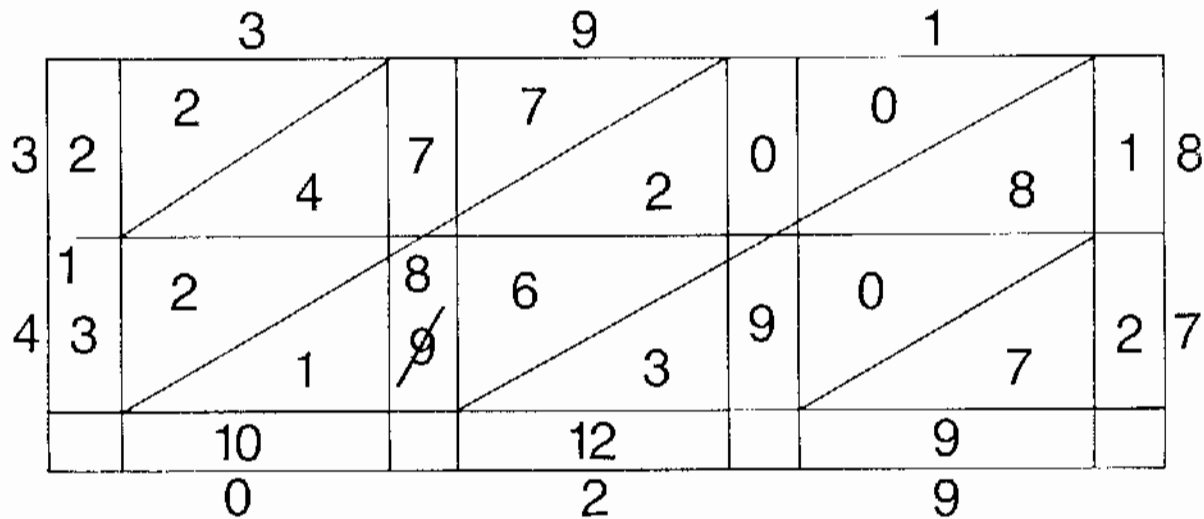


(b) Try a trial quotient of 3 for $340 \div 87$.



(c) Try a trial quotient of 9 for $792 \div 87$.

Fig. 4 continued



bove the 9. The 2 in the vertical column is subtracted from the 2 in the cell, leaving no remainder.

The same process is repeated using 12 (read down the second vertical strip and to the right) as the partial dividend and again using 90 as the trial divisor and 9 as the trial quotient figure (fig. 4(c)). Following the same procedure described previously ($9 \times 3 = 27$; $9 \times 7 = 63$; $2 - 3$ doesn't work, so borrow from the 9 in the vertical and write 12 in the horizontal: $6 \div 2 = 3$; $8 - 8 = 0$; $7 - 7 = 0$) leaves 09 in the remainder strip. Then 099 is the final partial dividend. In figure 4(d), we try a trial quotient of 1 for $099 \div 87$. The remainder, 12, is left in the strip at the far right. The quotient is 391 with a remainder of 12.

Lattice multiplication has always interested students. The extension into division offers an enrichment activity that can add depth to students' understanding of multiplication and division. Using these algorithms has also been successful in encouraging children to check their computations made originally with the standard algorithms.

Reference

Hutchings, Barton. "Low-Stress Algorithms." In *Measurement in School Mathematics*, 1976 Yearbook of the National Council of Teachers of Mathematics, p. 218-39. Reston, Va.: The Council, 1976. ■

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