POLYGONS THAT TESSELLATE

Regular Polygons

There are only three *regular* polygons that can tessellate a plane without being combined with other polygons. They are (1) equilateral triangles, (2) squares, and (3) regular hexagons. The following illustrations show this to be true.

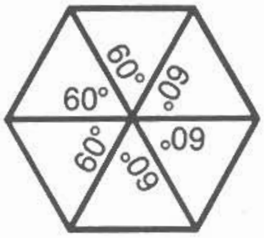


Figure 1-18
Equilateral Triangles
$6 \times 60° = 360°$

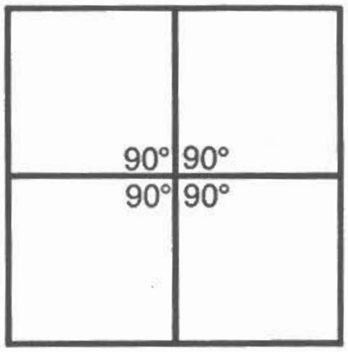
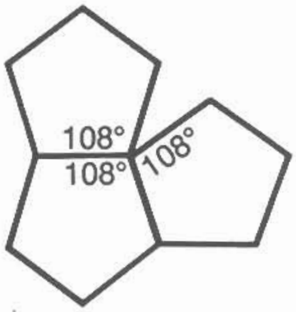


Figure 1-19
Squares
$4 \times 90° = 360°$



Does not tessellate

Figure 1-20
Pentagons
$3 \times 108° < 360°$
$4 \times 108° > 360°$

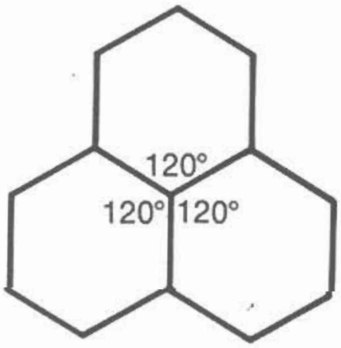
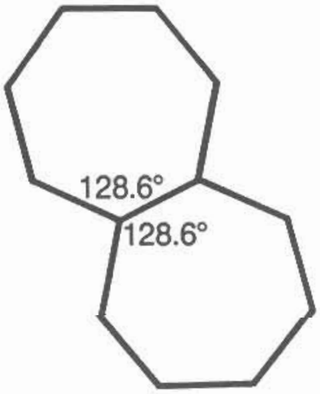
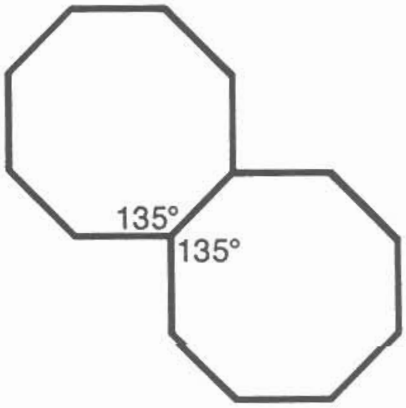


Figure 1-21
Hexagons
$3 \times 120° = 360°$



Does not tessellate

Figure 1-22
Heptagons
$2 \times 128+° < 360°$
$3 \times 128+° > 360°$



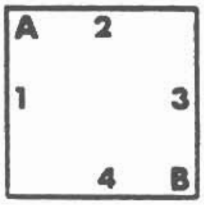
Does not tessellate

Figure 1-23
Octagons
$2 \times 135° < 360°$
$3 \times 135° > 360°$

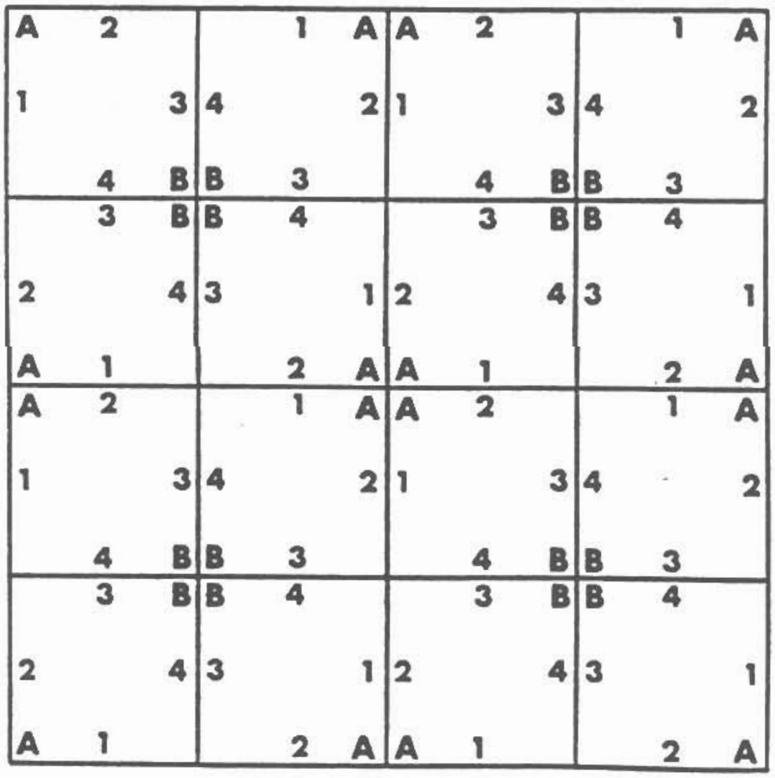
DESIGNING ESCHER-LIKE PATTERNS

It should now be evident that it is the underlying polygon tessellation and the relationships that exist between the congruent polygons in the tessellation that are of prime importance in designing Escher-like tessellations.

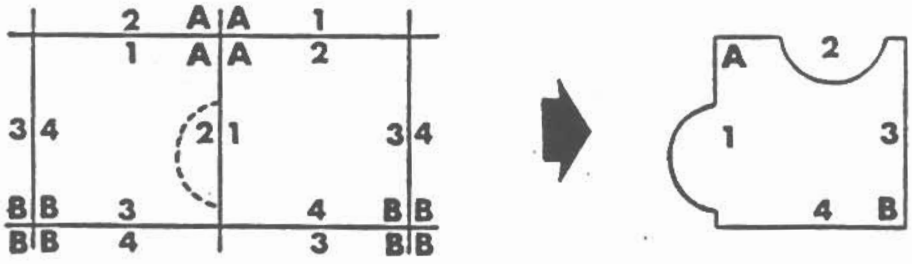
Let us select a polygon which tessellates the plane. A square is a convenient choice. Now investigate different 'procedures' which could be used to generate the tessellation from this single square. As one example, if we label the square as follows:



then the square could be rotated four times about B, 90° each time, before it returns to its original position. It could then be translated two squares over, both horizontally and vertically, and the rotations repeated. If we keep track of where the numbered sides land as it moves over the plane, we obtain the following:



If we examine the arrangement on the preceding page, we see that a bump on side 1 is a congruent hole on side 2, both equidistant from point A (and vice versa).

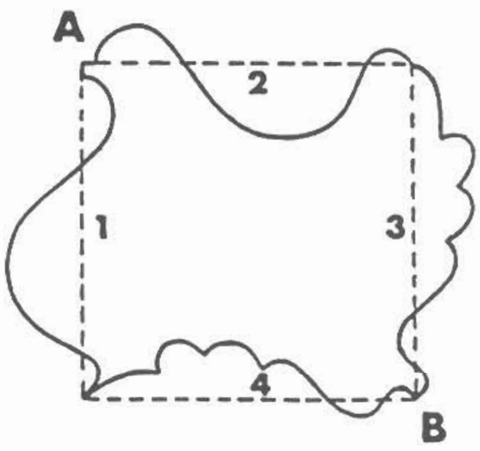


Sides 3 and 4 are likewise related. We can summarize these relationships with the following notation:

1↔2 **Measure from A**

3↔4 **Measure from B**

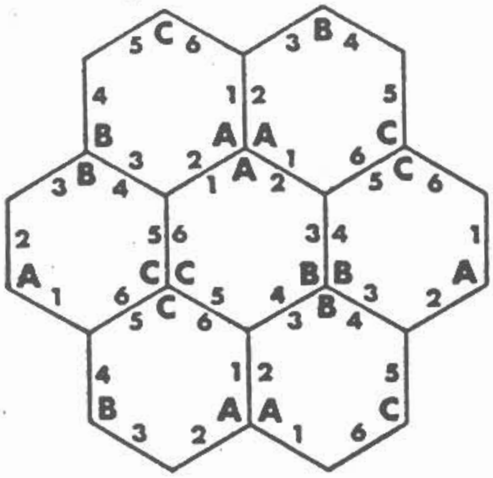
Now applying the rule several times in succession to one square we have:



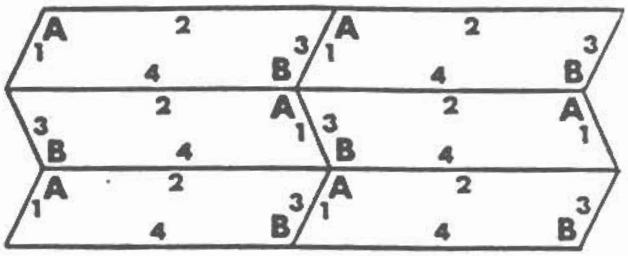
The modified square will tessellate the plane in the same manner as did the parent square. The completed tessellation is shown on p. 4 (Design ①).

Below are the schemata for design (2) and design (3) on p. 4 .

(2)



(3)



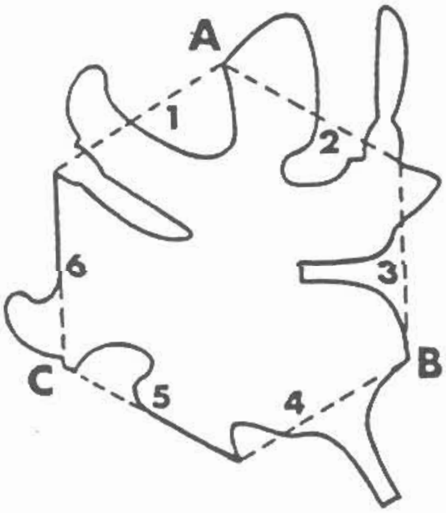
1↔2 Measure from A

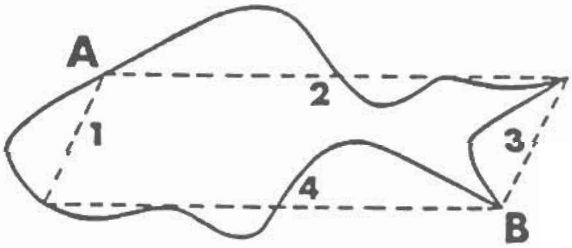
3↔4 Measure from B

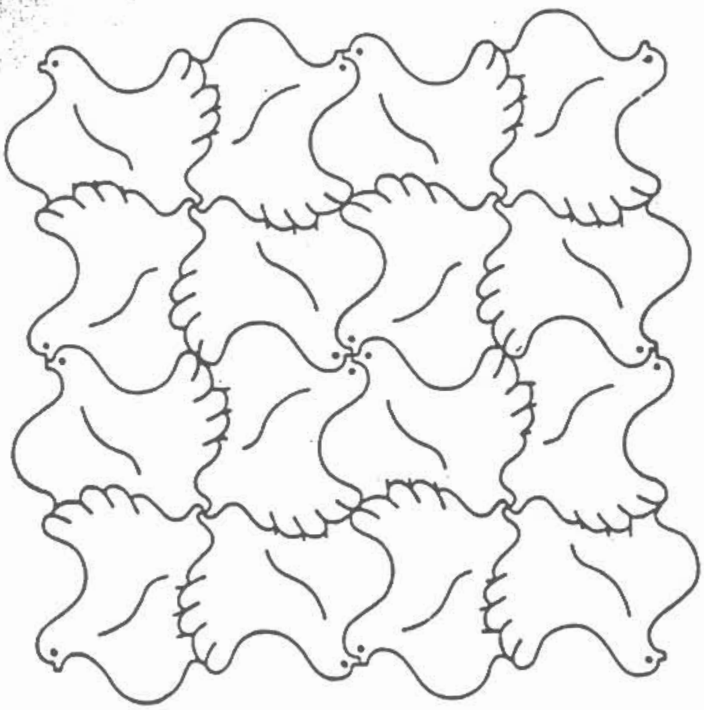
5↔6 Measure from C

1↔3 Translation

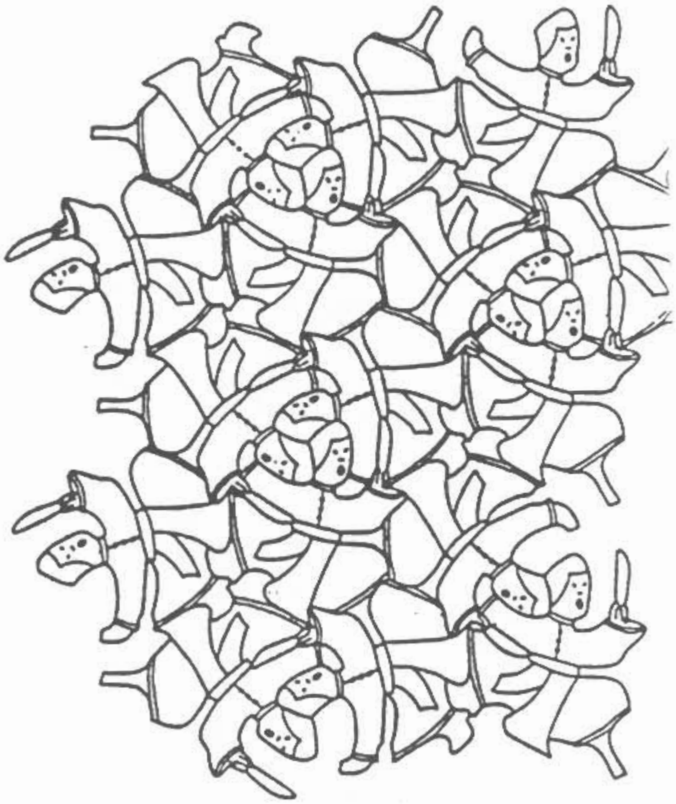
2↔4 Measure from A on 2 and from B on 4



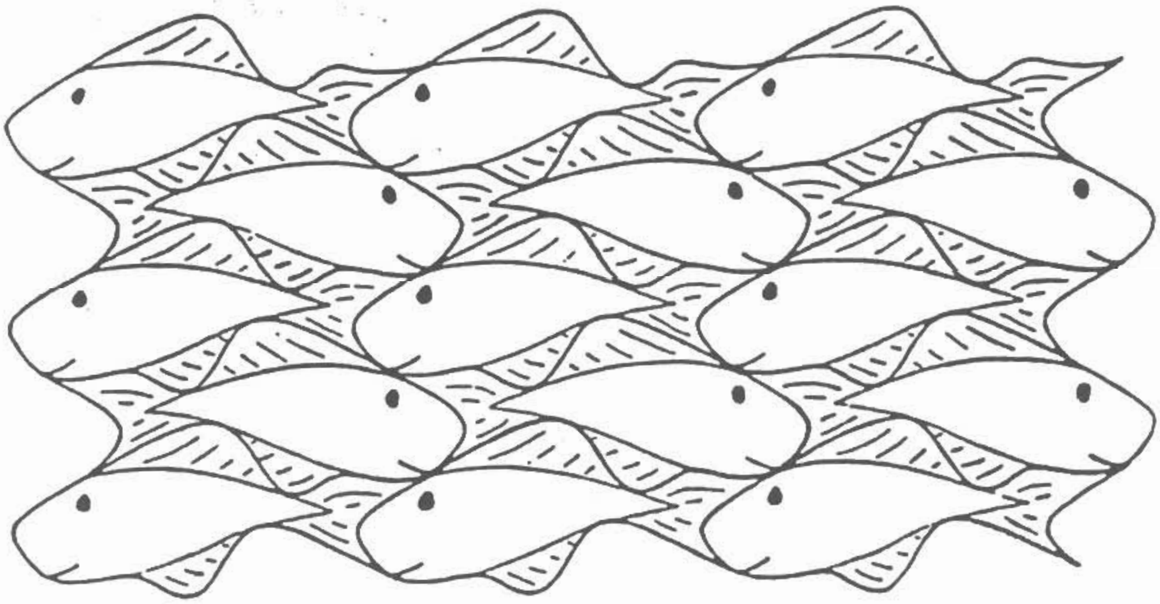




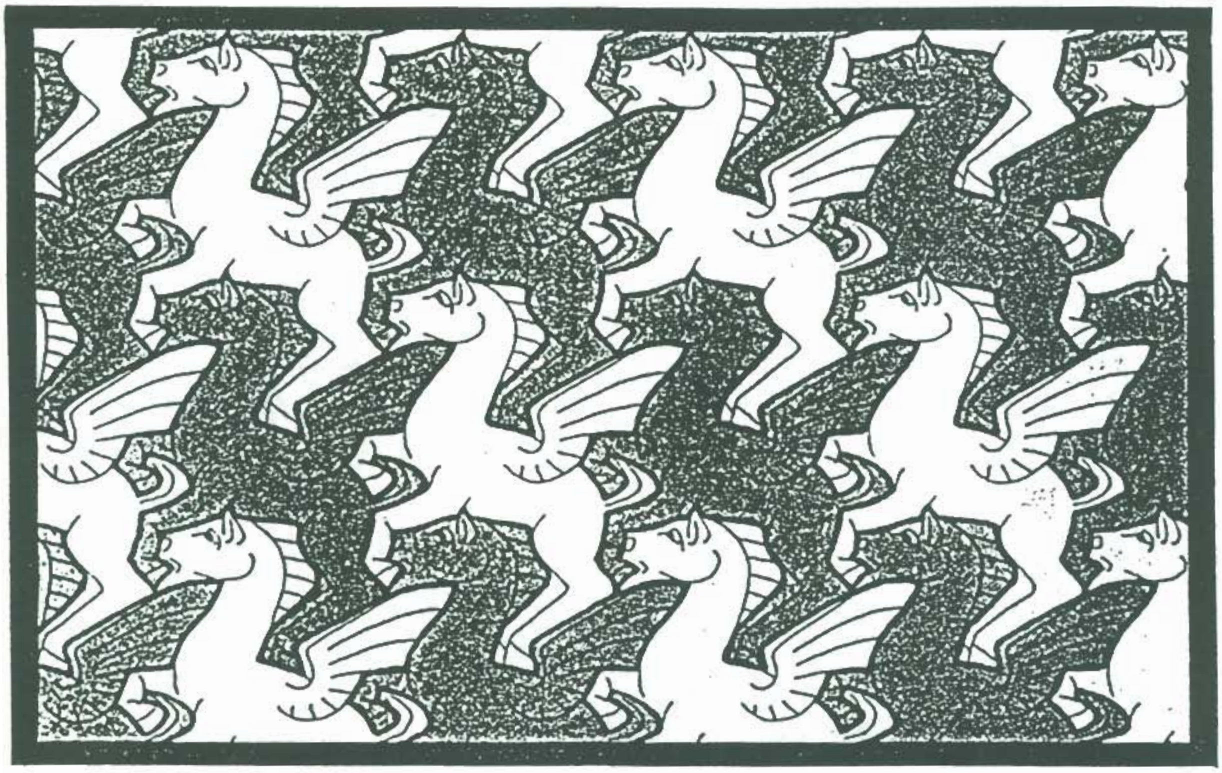
(1) Translation and Rotation (2-fold and 4-fold) Symmetry

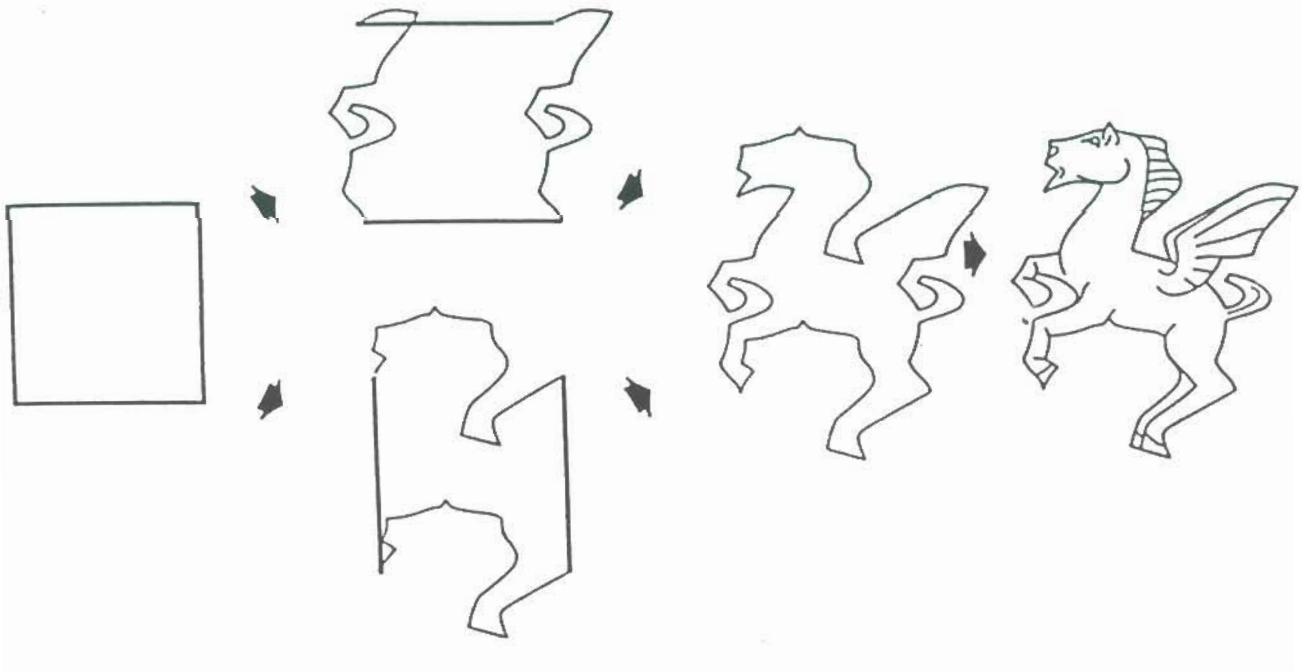


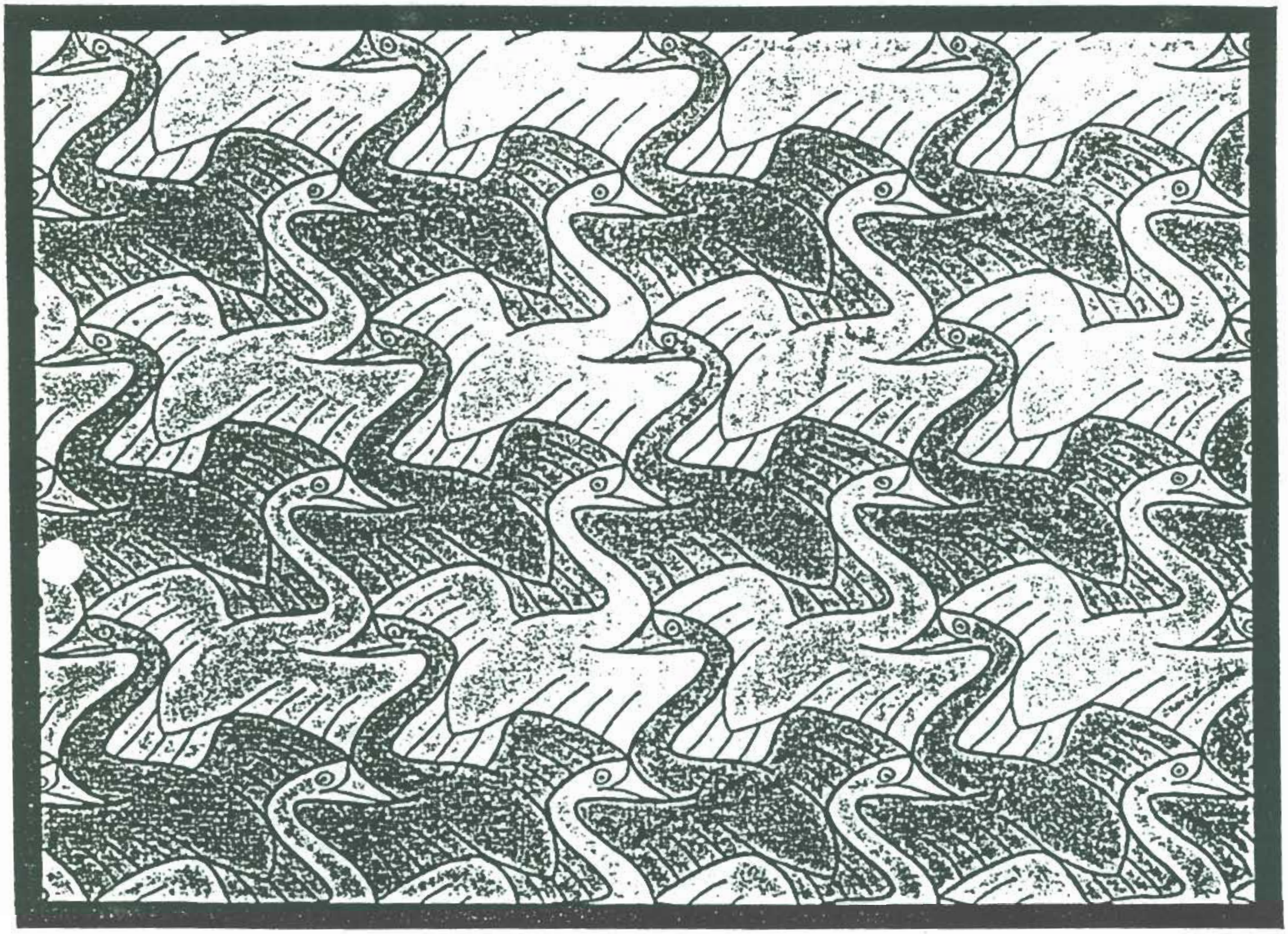
(2) Translation and Rotation (3-fold) Symmetry



(3) Translation and Glide-Reflection Symmetry







M. C. Escher. Study of Regular Division of the Plane with Birds.
India ink and watercolor, 1955

M. C. Escher. Study of Regular Division of the Plane with Reptiles. Pen, ink, and watercolor, 1939

Making a Tessellation

Introduction:

The purpose of this activity is to encourage teachers to undertake projects which involve topics unfamiliar to them. It is to be expected that most participants have seen tesselations, but fewer of the participants will have a working knowledge of them, and few, if any, will have ever designed their own.

The Topic Abstract secion contains information and sources for tessellation projects. The goal here is to demonstrate how easy it is to make a tessellation by using a stencil.

Discuss what a tessellation is: a shape which will cover a planar region without gaps and without overlapping. Simple examples would be the tiles covering a floor or ceiling. Regular polygons which tesselate the plane include triangles, squares and hexagons. *Show transparency that shows △ □ ⬠ etc w/ math below to prove they can/can't tesselate (int. ∠ must s. 360°*
Extend this by showing transparency 1. Explain briefly the process of making "holes" and "bumps" and the idea of conservation of area. Since the first example uses rectangles, show transparency 2 which uses parallelograms.

Conclude this portion by mentioning the resources available.

R

With 2 transparencies, show points of rotation

Activity: *$14.95*

Creating Escher-Type Tessellations
From Creative Publications

The activity planned for the workshop was inspired by George Escher, son of M. C. Escher. It is an activity which he did as a child, but he used potato stamps versus a stencil.

TRANSLATION
ROTATION

Show the top portion of transparency 3 which shows a 4 x 4 grid. The key to making the stencil is to always have the same length opening at the same distance from each vertex. It is also desirable to avoid any symmetry in the stencil. Demonstrate a possible design.

Show the bottom portion of transparency 3. This is the design which has already been copied for the participants. Explain that each of them will need to cut out the design. Demonstrate by placing a pre-cut stencil on the overhead. Distribute the acetate sheets, razor blades and cardboard. Have each participant cut out their stencil.

Now discuss how that one stencil can be used to tesselate the plane. Demonstrate the eight possible positions. Show transparency 4 which summarizes the positions and distribute the same summary sheet to the participants along with a sheet of plain paper and crayons (or markers).

The participants are going to create a tesselation by selecting any 4 of the 8 possible positions of the stencil. This will be denoted using the following notation:

example:

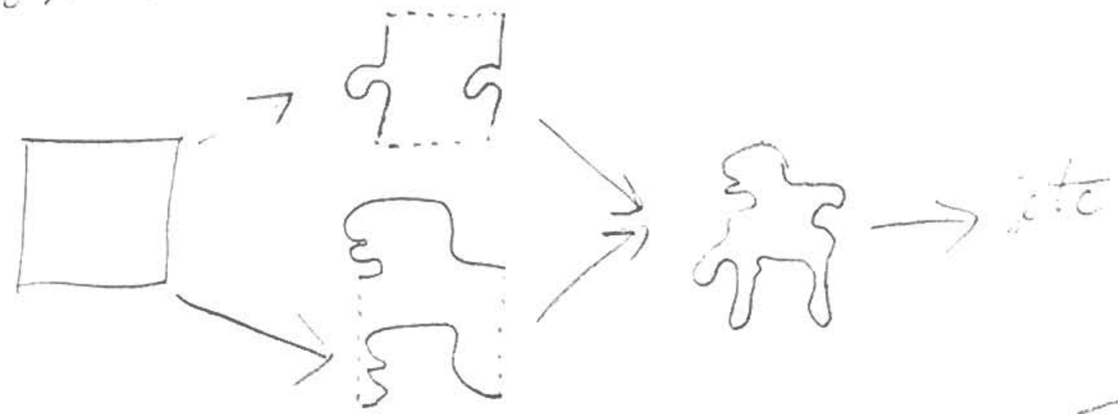
1	2
3	4

Demonstrate on the overhead what this tesselation would look like. Now the participants should select the 4 positions which they want to work with and begin their tesselation.

When they have finished, discuss the following points:

- Selecting 4 positions was purely arbitrary. Any number of positions, repeated sequentially, will tesselate the plane.
- The collection of tesselations by the group could now be used to tesselate a bulletin board.
- Six of the stencil designs could be folded to form a cube.
- How would this project be graded? Ask for participant input.

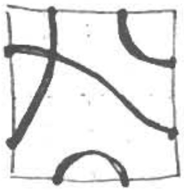
To make the horse



Conservation of area

over

We know the square will tessellate



go in one square and a little bit -- connect with any design (Be sure the lines are the <u>same</u> thickness)

Put on an acetate & cut out to make a stencil.
(Use <u>heavy</u> acetate)

There are 8 different positions (4 90° rotations &
4 flip side ")

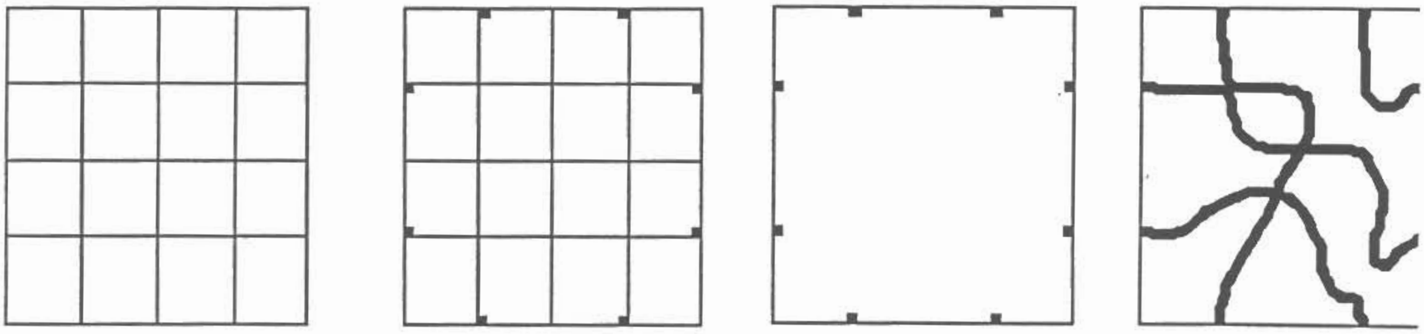
Decide on a pattern -- create your design

Why tessellations?

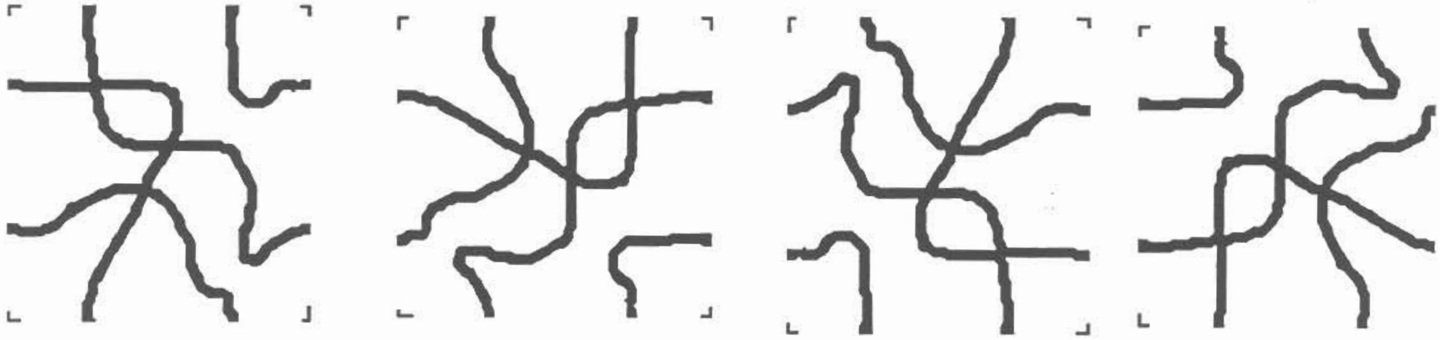
① transformational geometry
- reflections
- rotations
- translations

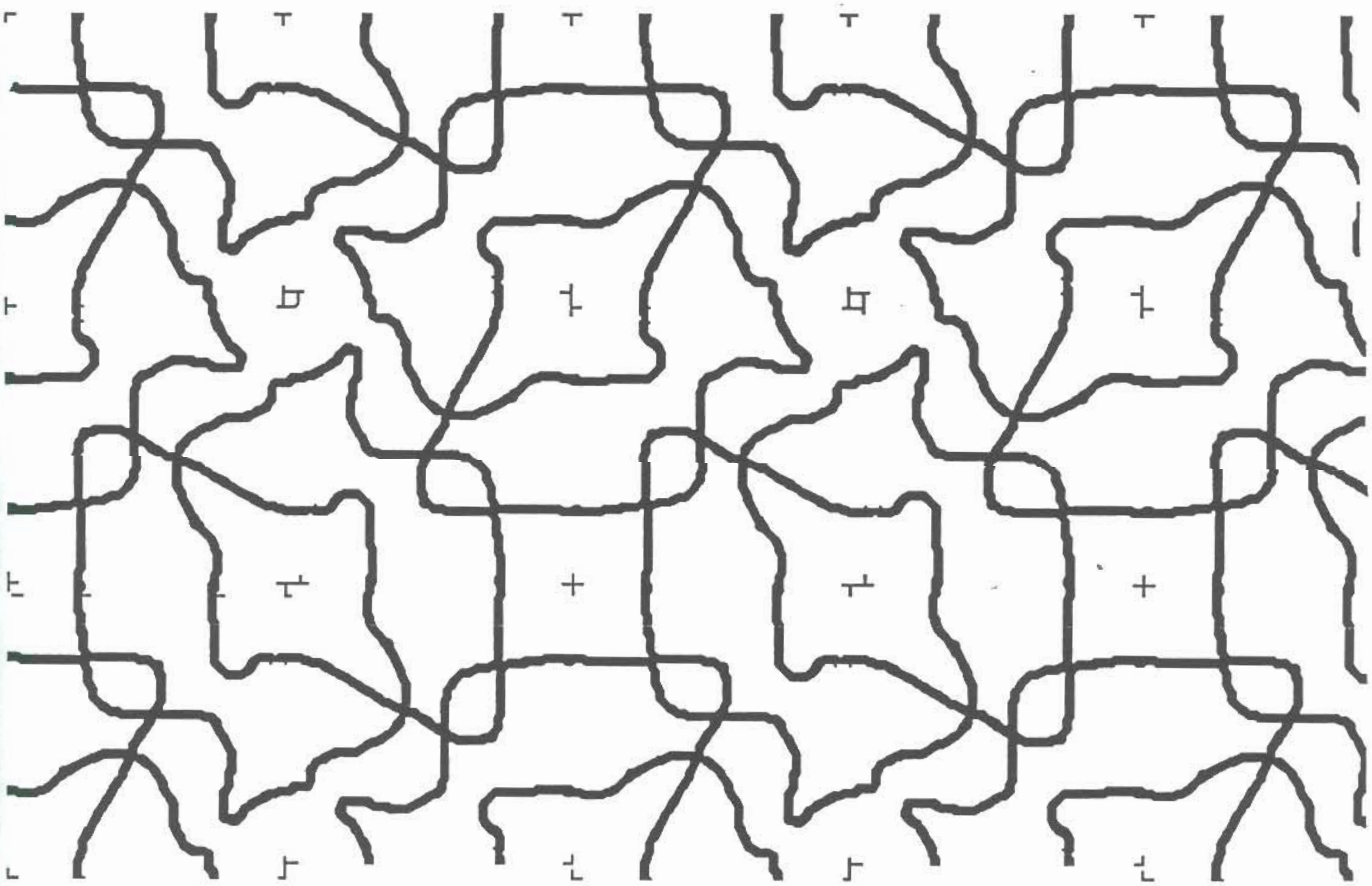
2) area

2 1/8



1 2 3 4





TESSELLATIONS