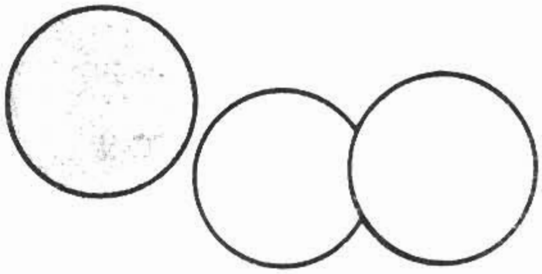
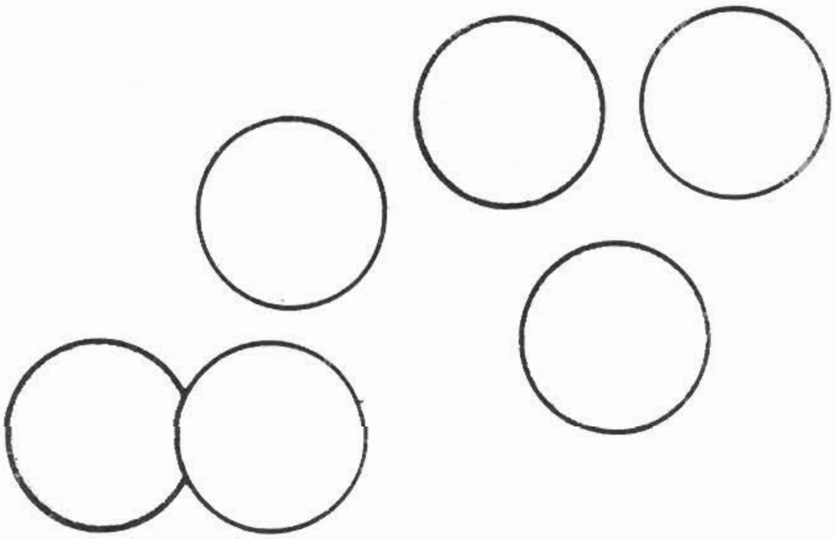
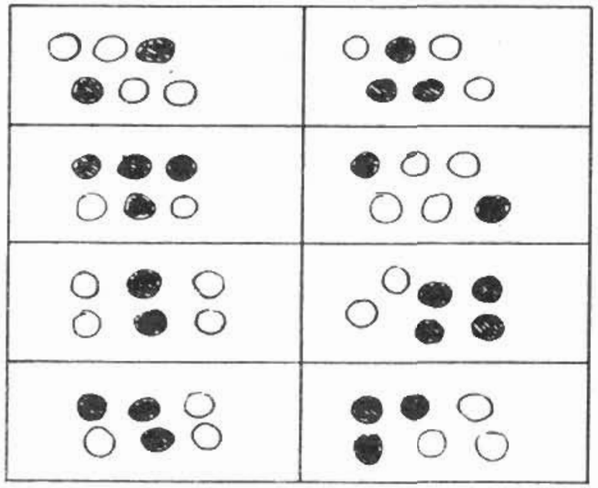


Two-Color Counters



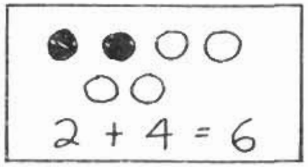
III EXPLORING ADDITION COMBINATIONS

Children can use Two-Color Counters to explore the combinations of numbers up to ten. They will need counters, red and yellow crayons, and a worksheet for the number you've assigned (see sample below). You assign a number; for example, 6. Children each take 6 counters and spill them carefully on the table, recording each spill by coloring on a worksheet. (Make worksheets for each of the numbers from 4 to 10 to have on hand.)



Sample worksheet for 6

When children complete their sheets, show them how they can record the corresponding addition equation in each box. For example:



IV EXPLORING SUBTRACTION COMBINATIONS

Two-Color Counters can also help children link subtraction equations to a concrete material. Assign a number; 8, for example. Children each take 8 counters, drop them, and sort by color. They slide the reds to one side and then record a subtraction equation, showing what they have done.

EXAMPLE: 8 – 3 = 5

V HOW MANY MORE?

It is difficult for young children, when comparing two sets of objects, to figure "how many more" there are in one set than in another. They need many concrete experiences to make sense out of this language pattern. The Two-Color Counters are useful for this. Working with a small group, have one child take a handful of counters, sort them by color, and arrange them in two rows. Ask questions to the group: "How many reds? How many yellows? How many more ________ will you need so both rows have the same?"

This will be difficult for some children. Do not make it a testing experience, but instead, one in which the children can experience the concept over time.

VI NUMBER RIDDLES

The children use Two-Color Counters to solve riddles that focus on number relationships. The following are examples to use. Children should show the solutions with the appropriate counters.

"I have 4 counters in all. There are 2 more reds than yellows."

"I have 11 counters in all. I have 1 more yellow than red."

"I have twice as many reds as yellows. Altogether, I have 9 counters."

"There are 2 more reds than yellows. I have a dozen counters in all."

EXTENSION:
Include some riddles for which there can be more than one right answer.

"I have 12 counters in all. There are more reds than yellows."

"I have 10 counters altogeter. I have an even number of each."

"I have 2 more red than yellow counters. I have less than 20 counters in all."

Students working in small cooperative groups can try to find all possible solutions for these types of riddles and record them.

FURTHER EXTENSION:
Encourage children to write their own riddles after they have had sufficient experience. If they write each riddle on an index card, they will contribute to a class set of puzzle cards.

VII INTRODUCING FRACTIONS

Two-Color Counters are useful for presenting beginning fraction concepts. For example, to develop the concept of halves as two equal parts of a set, ask children to display 4 yellow counters. Ask them to turn some counters to show half red and half yellow. Encourage children to explain why they flipped two counters. ("Because 2 and 2 are 4." "Because both halves have to be the same,", etc.) Define one half as one of two equal parts. Repeat the activity, having them display 6 yellows, then flipping the counters to show half and half; repeat with 8, 10, etc. Have them try an odd number, 7, for example, and discuss why this is not possible. Develop the generalization that for any set with an even number of counters, two halves can be formed when both sets contain exactly the same quantity.

Continue with the concept of thirds. Ask children to show a set of 6 red counters and separate them into three equal groups. Introduce the term one-third to describe each group. Define a third as one out of three equal parts. Direct them to flip one-third to yellow. Ask: "How many of your set is red? What fractional part is red?" Repeat with 9 counters; 12 counters. Develop the generalization that for any set, three-thirds may be formed when all three sets contain exactly the same quantity.

Repeat the activity to develop the concept of fourths (one out of four equal parts), using 8, 12, and 16 counters.

Notice that no symbols were introduced with this activity. When developing a concept, symbols can interfere with the learning. The activity explains how to connect the concept to the appropriate symbols.

VIII IDENTIFYING FRACTIONS OF A SET

Have each child take 6 counters. Explain that the 6 counters form one whole set. Ask the children to spill their counters carefully and sort them by color. Then have each, in turn, tell what part of the set is yellow and what part is red. Encourage children to be specific and to use accurate vocabulary.

"5 out of 6 are yellow; 1 out of 6 is red."

"3 counters in the set are yellow and 3 are red.", etc.

Introduce fraction symbols that connect the children's verbalizations to the correct symbolization.

$\frac{6}{6}$ = the whole set $\frac{5}{6}$ = yellow $\frac{1}{6}$ = red

$\frac{6}{6}$ = the whole set $\frac{3}{6}$ = yellow $\frac{3}{6}$ = red

Ask children to demonstrate the symbols with the actual counters.

Form a new set by taking 8 counters. Repeat the above activity several times to find all possible fractions of the set. Help children to organize their findings by using a chart.

Drawing	Total in Set	Fraction Red	Fraction Yellow
	$\frac{8}{8}$	$\frac{1}{8}$	$\frac{7}{8}$
	$\frac{8}{8}$	$\frac{2}{8}$	$\frac{6}{8}$

Repeat this activity, varying the number of counters in the set.

IX FLIPPING FRACTIONS

Ask children to show 6 red counters. Then ask them to flip some so that 1/2 are yellow. Model how to record this on the chalkboard: 1/2 of 6 = 3. Have children turn all the counters back to red and then flip them so the 1/3 are yellow. Record on the chalkboard: 1/3 of 6 = 2. Continue so that children flip the counters to show 2/3 yellow (4 yellow counters), 1/6 yellow (1 counter); 4/6 yellow (4 counters); etc. Record each on the chalkboard.

Repeat the activity with 12 yellow counters. Ask children to flip the counters to show: 1/2 red (6 counters); 1/3 red (4 counters); 1/4 red (3 counters); 2/3 red (8 counters); 3/4 red (9 counters); etc. Record each action on the chalkboard (i.e., 1/2 of 12 = 6; 2/3 of 12 = 8, etc.).

Follow up with an independent activity for children to work on in pairs. A worksheet can be used as shown. Change the number to vary the experience.

> Flipping Fractions
>
> Use __8__ counters.
>
> Write all the fractional sentences you can.
> For Example: 1/2 of 8 = 4.

X FRACTION RIDDLES

The children use Two-Color Counters to solve fraction riddles. The following are examples to use. Encourage children to write their own riddles after sufficient experience.

"I have 6 counters. Two-thirds of them are red."

"I have a set of counters. Nine of them are red and 1/3 that number are yellow."

"I have a set of counters with 1/2 yellow and 2/4 red." (Many solutions are possible.)

XI PROBABILITY TOSS

The Two-Color Counters provide an effective model for introducing basic concepts in probability and statistics. If a counter were tossed many times, as with a coin, it should land with the red side up half the time and the yellow side up half the time. Though this 50-50 result may not happen exactly, the more times a counter is tossed, the closer the results should be to a 50-50 distribution. Test this with the class by having students work in pairs. Each pair tosses a counter 10 times, keeps track of how many times each color landed face up, and records their results on a class chart or the chalkboard.

											TOTALS
Red	7	5	4	3							
Yellow	3	5	6	7							

Students can add the numbers. (A calculator is useful for this.) Discuss the results. If the students are interested, have each pair do an additional 10 tosses, add the data on to the existing chart, and compare.

XII THE TWO-COUNTER EXPERIMENT

Present this question to the class: "If you toss 2 counters, what are the possible results?" The students need to see that three possible outcomes can result:

1. Two reds 2. Two yellows 3. One red and one yellow

Present another question: "If we tossed 2 counters many times and kept track of how many times each possible outcome occurred, what do you think would happen?" You may want students to write their predictions.

Then, as in Activity XI, students work in pairs tossing two counters 10 times. Again, they record their results on a class chart.

											TOTALS
Two Reds											
Two Yellows											
One Red, Yellow											

Students add the numbers and discuss the totals. They should repeat for another 10 tosses for each pair and adjust the statistics.

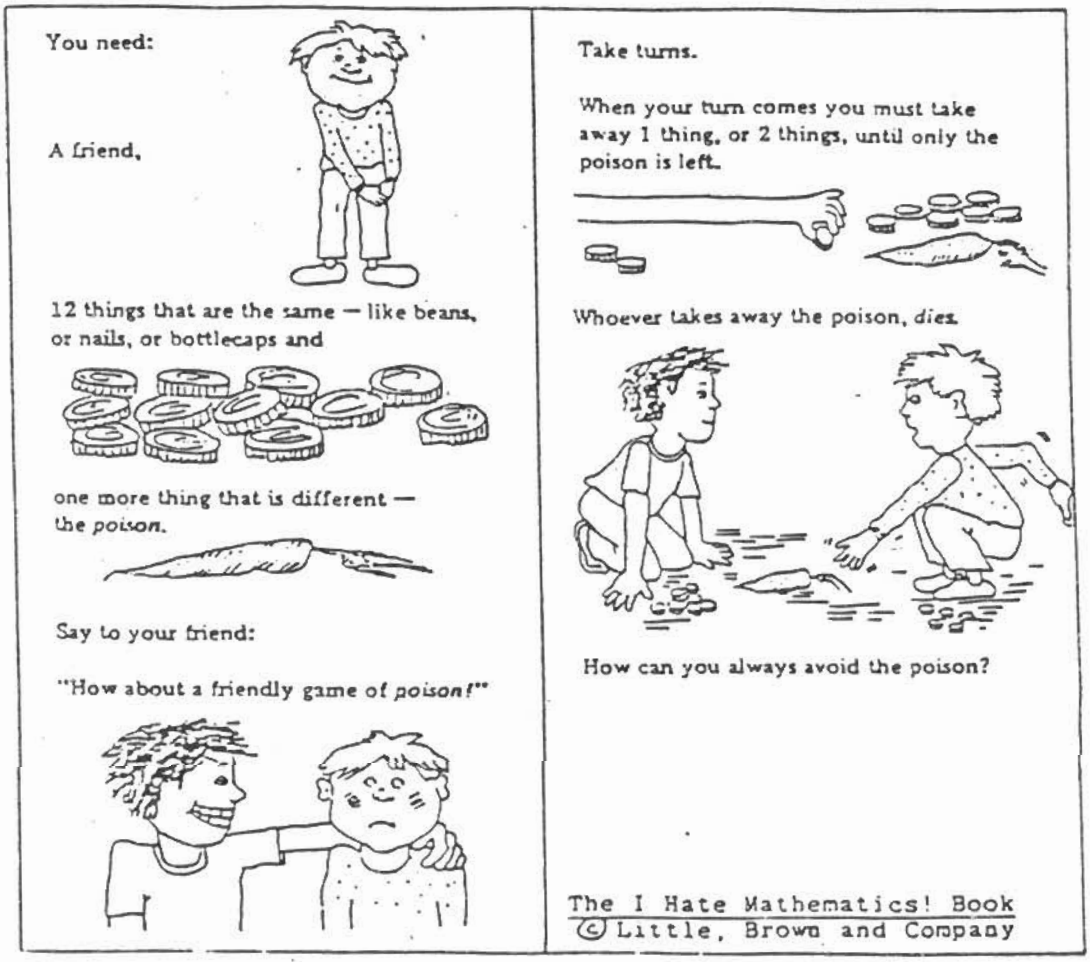
What will result is that the outcome of "one red and one yellow" will occur much more often. This is because though there are three possible outcomes, they are not equally likely. There are two different ways that "one red and one yellow" can occur. If you marked one counter #1 and the other #2, then sometimes it will be #1 that comes up red while #2 comes up yellow. Other times it will be reversed. This means that there are four equally likely ways for the counters to produce the three possible outcomes.

	Counter #1	Counter #2
Two Red	Red	Red
Two Yellow	Yellow	Yellow
One Red and One Yellow	Red	Yellow
	Yellow	Red

NIM-TYPE GAMES

These are a collection of two-person games that are all similar, even though they may not seem so. They can be played by children (and adults) of all ages. But to explore all aspects of each game can be a good challenge even for the more abstract thinkers. When introducing them to children (and just one at a time!), they need a chance to just play the game to get a feel for it. Then I ask them (if they're old enough) to start a list of strategies they notice that help them win. I try to get pairs of students to work cooperatively on the strategies, rather than purely on the winning. A class chart of strategies helps to spread the thinking around the room.

1. POISON



The I Hate Mathematics! Book
© Little, Brown and Company

Variations on the game:

1. What would happen if you used a different number of objects? Investigate to see how this would change your strategies.

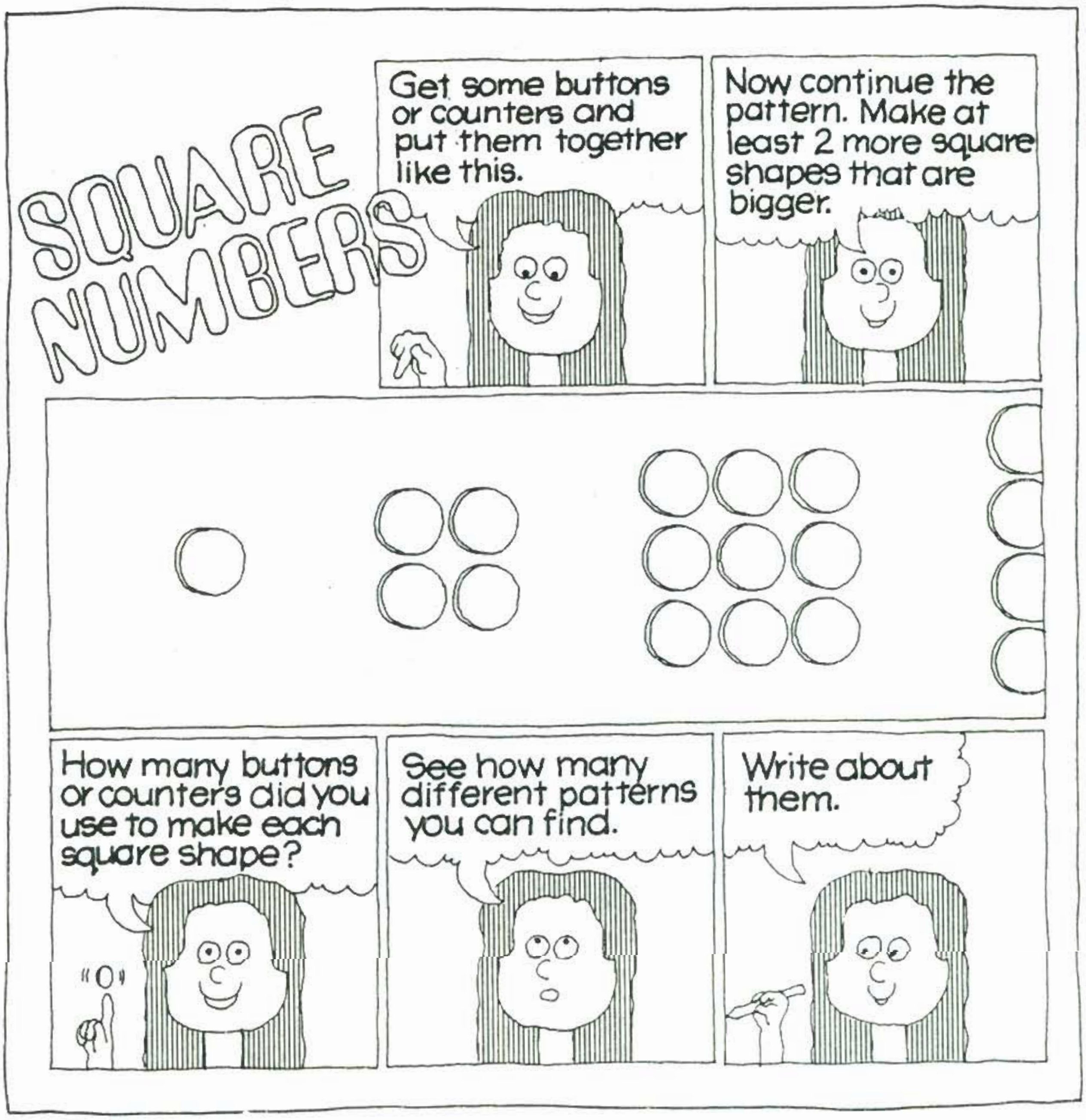
2. Change the rules so you can take away 1, 2, or 3 objects. Try this with 13 objects, and then change that number when you're ready to investigate further.

Event

59

Functions and Graphs

★



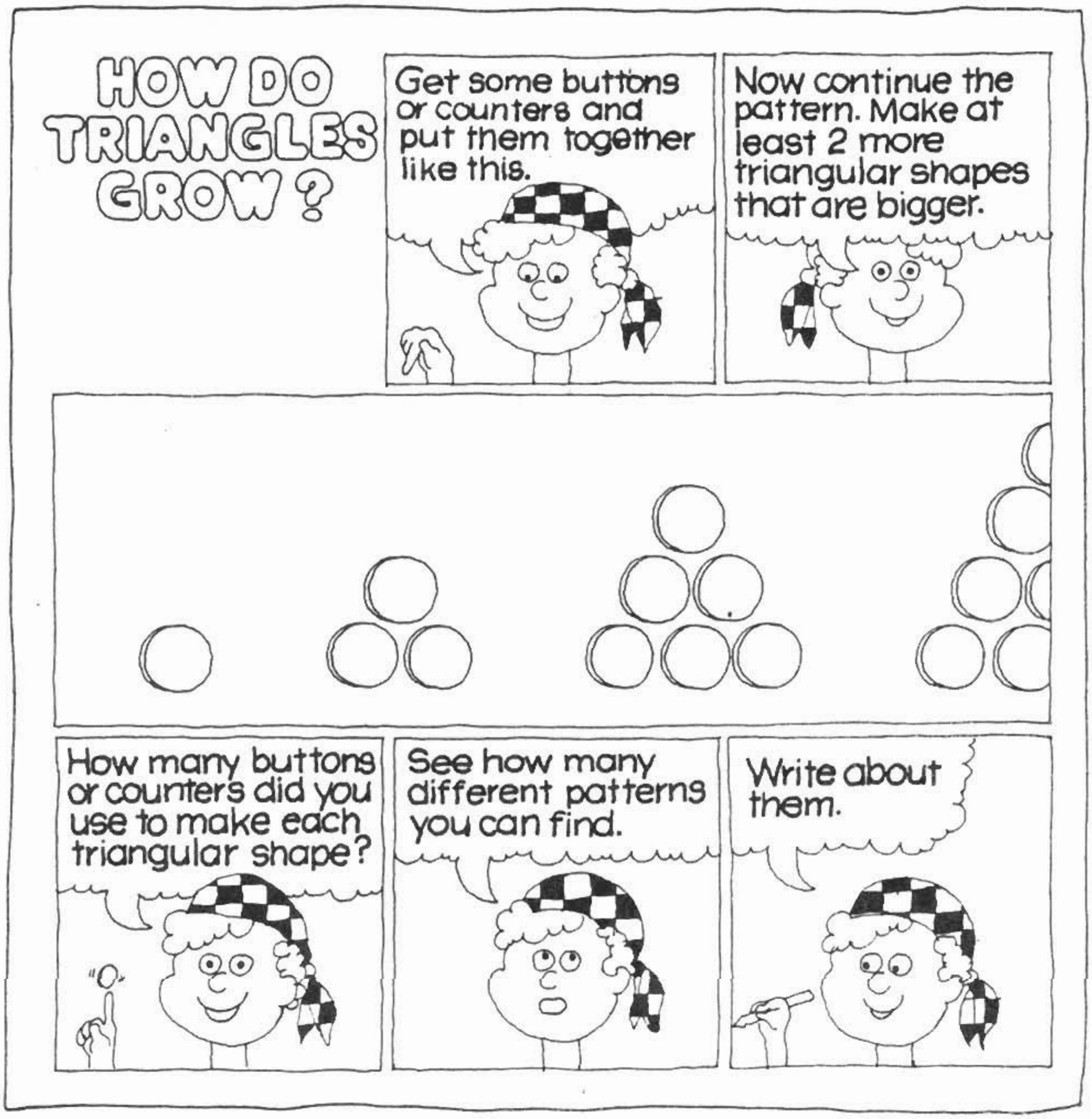
Seeing patterns is an important part of mathematics. In this activity, you investigate number patterns which come from shapes that grow.

Event

64

Functions and Graphs

★★



Seeing patterns is an important part of mathematics. In this activity, you investigate number patterns which come from shapes that grow.